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METHOD OF NONDESTRUCTIVE CONTROL OF THE THERMOPHYSICAL PROPERTIES OF ROCKS ON BOREHOLE CORES

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The problem of the temperature field of a semi-infinite cylinder with a heat source of constant power in the form of a round contact spot acting on its end-face surface has been solved. A method of nondestructive control of the thermophysical properties of rocks on borehole cores is suggested.

Introduction. Recently, the methods of nondestructive control of the thermophysical properties of bulky materials have found widespread application in the practice of thermal measurements. Their theory is based on the laws governing temperature perturbations in a semi-infinite space from a heat source in the form of a circle on its surface.

It is of interest to investigate the possibilities of using these methods for measuring the thermophysical properties of rock samples from borehole cores that are characterized by finite diameters. For this purpose, it is necessary to solve the problem for a finite region having, in particular, the shape of a cylinder. Various variants of the problem were considered in [1]. The two-dimensional temperature field of a finite cylinder with one of its end-faces heated by a local heat source of circular shape or in the form of a ring with time-variable power and temperature, when on other surfaces of the body the boundary condition of the third kind is given, has been studied. The latter is the generalized case of heat transfer of a body with the environment, since many particular cases of heat transfer may follow from it. The solutions of the problem obtained in [1] are of theoretical interest in the development of various variants of the method of nondestructive control of the thermophysical properties of materials in conformity with the technical possibilities of realization of theoretically adopted conditions of heat exchange of a body with its environment. Here, we restrict ourselves to a particular problem in application to borehole cores of rocks (there are no limitations on the length).

Statement of the Problem. We have a cylinder of infinite length of radius R. On the end-face surface of the cylinder ($z = 0$) there is a heat source of constant power Q, of radius r_0, $0 < r_0 \le R$. The end-face ($z = 0$) and side ($r = R$) surfaces of the cylinder are insulated. The initial temperature of the cylinder is constant and equal to the temperature of the environment t_{en}. At a large distance along the length ($z \to \infty$) the temperature of the cylinder does not change in the period of heat source action. The problem is axisymmetric.

The problem is formulated mathematically as follows:

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{a}\frac{\partial t}{\partial \tau} \quad (0 < z < \infty\,,\ \ 0 < r \le R)\,, \tag{1}$$

$$t(r, z, 0) = t(r, \infty, \tau) = t_{en}\,, \tag{2}$$

at any z

$$\frac{\partial t(R, z, \tau)}{\partial r} = 0\,, \tag{3}$$

at $z = 0$

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$$\frac{\partial t\,(r, 0, \tau)}{\partial z} = \begin{cases} -\frac{q}{\lambda} = \text{const} & \text{for} \quad 0 \le r \le r_0 \,, \quad \text{where} \quad q = Q/\pi r_0^2 \,, \\ 0 & \text{for} \quad r_0 \le r \le R \,, \end{cases} \tag{4}$$

at $r = 0$

$$\frac{\partial t\,(0, z, \tau)}{\partial r} = 0 \quad \text{and} \quad t\,(0, z, \tau) \ne \infty \,. \tag{5}$$

We introduce the notation $\vartheta(r, z, \tau) = t(r, z, \tau) - t_{\text{en}}$. Then, according to (2) $\vartheta = 0$, the remaining conditions for ϑ are the same as for t.

Solution of the Problem. We will seek the solution by successive application of the Hankel and Laplace methods of integral transformations:

$$\bar{T}_{\text{H}}\,(p, z, \tau) = \int_0^R r\vartheta\,(r, z, \tau)\, J_0\,(pr)\, dr \,, \tag{6}$$

$$\bar{\theta}_{\text{H}s}\,(p, z, s) = \int_0^\infty \bar{T}_{\text{H}}\,(p, z, \tau) \exp\,(-s\tau)\, d\tau \,, \tag{7}$$

where p is the root of the characteristic equation

$$J_0'\,(pR) = 0 \,. \tag{8}$$

In the notation of the Laplace–Hankel transformation $\bar{\theta}_{\text{H}s}$ the system of equations (1)–(5) may be presented in the form

$$\frac{\partial \bar{\theta}_{\text{H}s}}{\partial z^2} - \left(p^2 + \frac{s}{a}\right)\bar{\theta}_{\text{H}s} = 0 \tag{9}$$

under the boundary conditions:
for $z = 0$ and any r

$$\frac{\partial \bar{\theta}_{\text{H}s}\,(p, 0, s)}{\partial z} = -\frac{q r_0}{\lambda p s} J_1\,(p r_0) \,, \tag{10}$$

for $r = 0$ and any z

$$\frac{\partial \bar{\theta}_{\text{H}s}\,(0, z, s)}{\partial r} = 0 \,, \tag{11}$$

for $r = 0$ and $z \to \infty$

$$\bar{\theta}_{\text{H}s}\,(0, \infty, s) \ne \infty \,, \tag{12}$$

for any r and $z \to \infty$

$$\bar{\theta}_{\text{H}s}\,(p, \infty, s) = 0 \,. \tag{13}$$

Here, we obtain the following solution of the problem in transforms:

$$\bar{\theta}_{Hs}(p, z, s) = \frac{qr_0 J_1(pr_0)}{\lambda ps\sqrt{p^2 + s/a}} \exp\left(-z\sqrt{p^2 + s/a}\right). \tag{14}$$

Using the inversion theorem, we will successively go over from the transform (14) to the inverse transform, first following the Hankel transformation:

$$\bar{T}_H(p, z, \tau) = \frac{2}{R^2}\bar{\theta}_{Hs}(0, z, s) + \frac{2}{R^2}\sum_{n=1}^{\infty}\frac{\bar{\theta}_{Hs}(p_n, z, s) J_0(p_n r)}{J_0^2(p_n R)}, \tag{15}$$

where p_n are the roots of the characteristic equation (8). Having substituted the expression of $\bar{\theta}_{Hs}$ from Eq. (14) into Eq. (15), we obtain

$$\bar{T}_H(p, z, \tau) = \frac{q\sqrt{a}\, r_0^2}{\lambda R^2}\frac{\exp(-z\sqrt{s/a})}{s\sqrt{s}} + \frac{2qr_0}{\lambda R^2}\sum_{n=1}^{\infty}\frac{J_1(p_n r_0) J_0(p_n r)}{p_n J_0^2(p_n R)}\frac{\exp\left(-z\sqrt{p_n^2 + s/a}\right)}{s\sqrt{s + ap_n^2}}. \tag{16}$$

Using the tables of transforms with respect to s for Eq. (16), the solution of the problem will be written finally in the form

$$t(r, z, \tau) - t_{en} = \frac{qr_0^2}{\lambda R^2}\left(2\sqrt{\frac{a\tau}{\pi}}\exp\left(-\frac{z^2}{4a\tau}\right) - z\,\mathrm{erfc}\frac{z}{2\sqrt{a\tau}}\right) + \frac{qr_0}{\lambda}\sum_{n=1}^{\infty}\frac{J_1\left(\mu_n\frac{r_0}{R}\right)J_0\left(\mu_n\frac{r}{R}\right)\Psi_n(\mu_n, z, \tau)}{\mu_n^2 J_0^2(\mu_n)}, \tag{17}$$

where

$$\Psi_n(z, \tau, \mu_n) = \exp\left(-\frac{z\mu_n}{R}\right)\mathrm{erfc}\left(\frac{z}{2\sqrt{a\tau}} - \frac{\mu_n\sqrt{a\tau}}{R}\right) - \exp\left(\frac{z\mu_n}{R}\right)\mathrm{erfc}\left(\frac{z}{2\sqrt{a\tau}} + \frac{\mu_n\sqrt{a\tau}}{R}\right), \quad \mu_n = p_n R. \tag{18}$$

We will investigate this solution in the limiting cases.

1. If $z \to \infty$, then $\Psi_n = 0$ and $t - t_{en} = 0$, i.e., at a large distance from the body surface there are no changes in temperature.

2. If $r_0 = R$, then the sum of the series is equal to zero, since $J_1(\mu_n) = 0$; we will obtain the solution of the problem for a semi-infinite body heated from the surface by a power Q, i.e., at $q = Q/s$ = const (here $S = \pi R^2$ is the area of the end-face surface of the cylinder) [2]

$$t - t_{en} = \frac{2q}{\lambda}\left[\sqrt{\frac{a\tau}{\pi}}\exp\left(-\frac{z^2}{4a\tau}\right) - \frac{z}{2}\,\mathrm{erfc}\left(\frac{z}{2\sqrt{a\tau}}\right)\right]. \tag{19}$$

3. If $r_0 = 0$, then $(r^2/R^2) = 0$ and $J_1(0) = 0$. Then $t - t_{en} = 0$, i.e., there is no heating of the body.

4. If $R \to \infty$, then $(r_0^2/R^2) \to 0$, and the first term of Eq. (17) vanishes. In the second term the parameter μ_n assumes quite different values. Since the region considered is not limited at all over the radius, no limitations are imposed on the parameter μ_n and it assumes any values from 0 to ∞, and in the solution the sum is replaced by infinite integral over μ or $p = \mu/R$:

$$t(r, z, \tau) - t_{en} = \int_0^{\infty} c(p) J_0(pr)\Psi(z, \tau, p)\, dp, \tag{20}$$

which, in the case of boundary condition (4), assumes the form of Oosterkamp's solution [2]:

$$t(z, \tau, r) - t_{en} = \frac{qr_0}{2\lambda}\int_0^{\infty} J_1(pr_0) J_0(pr)\Psi(z, \tau, p)\frac{dp}{p}, \tag{21}$$

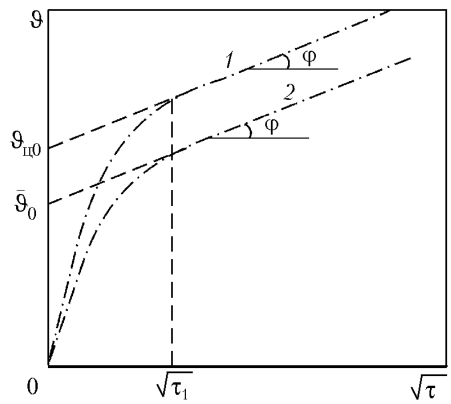

Fig. 1. Excess temperatures ϑ_c (1) and $\bar{\vartheta}$ (2) of the heating contact spot vs. the parameter $\sqrt{\tau}$. ϑ, °C; $\sqrt{\tau}$, sec$^{1/2}$.

where $c(p)$ is the integration parameter; $\Psi(z, \tau, p)$ is the same as in Eq. (18) but without the subscript n at p and Ψ. This corresponds to the case of heating a semi-infinite space through a contact spot of radius r_0 on its surface ($z = 0$).

Such a problem [3] can be met in calculations of the thermal resistance of contacting materials in the high-temperature thermophysics in different fields of technology, but here we restrict ourselves to the consideration of only stationary thermal state of bodies. However, from Eq. (17) it follows that such systems never attain a stationary thermal state.

Method Development. The solution (17) obtained allows one to develop the method of nondestructive control of the thermophysical properties of rocks on borehole cores. Here it is possible to use both the temperature of the heat source center and its integral mean temperature. We will consider this in more detail.

The temperature of the heat source ($z = 0$) is

$$\vartheta\,(r,\tau) = t\,(r,\tau) - t_{\text{en}} = \frac{2q}{\lambda}\left(\frac{r_0}{R}\right)^2 \sqrt{\frac{a}{\pi}}\,\sqrt{\tau} + \frac{2qr_0}{\lambda}\sum_{1}^{\infty}\frac{J_1\left(\mu_n\frac{r_0}{R}\right)J_0\left(\mu_n\frac{r}{R}\right)\operatorname{erf}\left(\frac{\mu_n\sqrt{a\tau}}{R}\right)}{\mu_n^2 J_0^2(\mu_n)}. \tag{22}$$

In the second part of Eq. (22) the error function erf rapidly tends to a constant value with time and at Fo ≥ 0.16 it becomes equal to 1 with an error of no more than 3%. Then at $\tau \geq 0.16R^2/a$ the following relation is valid: for the excess temperature of the heat source center ($r = 0$, $z = 0$)

$$\vartheta_c = t_c - t_{\text{en}} = N\sqrt{\tau} + \frac{2qr_0}{\lambda}B\,, \tag{23}$$

for the integral mean temperature of the heat source

$$\bar{\vartheta} = \bar{t} - t_{\text{en}} = N\sqrt{\tau} + \frac{4qR}{\lambda}K\,, \tag{24}$$

where N, B, and K are certain constants:

$$N = \frac{2q}{\lambda}\left(\frac{r_0}{R}\right)^2\sqrt{\frac{a}{\pi}}\,,\quad B = \sum_{1}^{\infty}\frac{J_1\left(\mu_n\frac{r_0}{R}\right)}{\mu_n^2 J_0^2(\mu_n)}\,,\quad K = \sum_{1}^{\infty}\frac{J_1^2\left(\mu_n\frac{r_0}{R}\right)}{\mu_n^3 J_0^2(\mu_n)}\,. \tag{25}$$

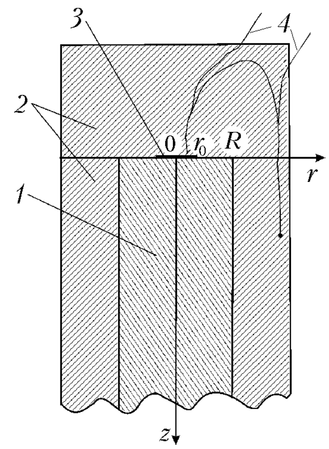

Fig. 2. The physical model of the system investigated and schematic diagram of the facility: 1) a test sample from the borehole core; 2) thermal insulation; 3) heating element; 4) differential thermocouple.

Whence it follows that with the experimental data being processed in the form of the plots of the functions $\vartheta_c = \sqrt{\tau}$ and $\vartheta = \sqrt{\tau}$ (Fig. 1), the latter in a certain interval of time $\tau_1 \geq 0.16R^2/a$ become linear, with the intersection of the straight lines with the ordinate axis ($\sqrt{\tau} = 0$) giving the values of ϑ_{c0} and ϑ_0 by which the thermal conductivity of the investigated sample is calculated from the following formulas:

$$\lambda = \frac{2qr_0B}{\vartheta_{c0}}, \tag{26}$$

$$\lambda = \frac{4qRK}{\vartheta_0}. \tag{27}$$

The thermal diffusivity of the sample is determined from the inclination angle φ of the plots of $\vartheta_c \to \sqrt{\tau}$ and $\vartheta \to \sqrt{\tau}$:

$$a = \frac{\pi\lambda^2 \tan^2 \varphi}{4q^2 \left(\frac{r_0}{R}\right)^4}. \tag{28}$$

The functions B and K have been tabulated depending on the parameter r_0/R.

The physical model of the system investigated and the schematic diagram of the facility are shown in Fig. 2. The basic elements of the facility are the heating element of circular shape 3 and a differential thermocouple 4 that records the excess temperature of the contact zone of heating (of the heat source). From the practical point of view it is convenient to assemble the heating element on a metal casing for effective averaging of temperature. One of the end-face surfaces of the sample 1 was smoothly ground. The sample is well insulated on all sides 2. The experiment is run at the needed thermostated constant temperature t_{en}.

It should be noted that in the developed dynamic mode of linear dependence of the excess temperature of the heat source on the parameter equal to the square root of time the distortions of the temperature field of the sample because of the heat capacity of the heater are excluded, which is an advantage of the proposed method of measuring the thermophysical properties of rocks on borehole cores. Moreover, the use of this method ensures, as compared to other well-known methods, a more accurate determination of the thermophysical properties of rocks, since the factor of finite dimensions of a sample undesirable for previous methods underlies the very theoretical foundation of the proposed method reducing to a minimum the error caused by the deviation of actually recorded temperature field from that adopted theoretically.

Conclusions. A method of nondestructive control of the thermophysical properties of rocks on borehole cores has been developed. It differs from those known by a factor that takes into account the boundedness of the dimensions (diameter) of a sample having been put into its theoretical basis.

The above-presented material has been submitted to the Federal Institute of Industrial Property (FIIP) as a request for a Russian Federation patent on "A Method of Nondestructive Measurement of the Thermophysical Properties of Rocks on Borehole Cores" (registration number 2006121332 of 15.06.2006).

NOTATION

a, thermal diffusivity, m^2/sec; $Fo = a\tau/R^2$, Fourier number; $J_0(pr)$, $J_1(pr)$, Bessel functions of zero and first order; p, parameter of Hankel integral transformation $\overline{T}_H$; Q, thermal capacity of a heater, W; q, heat flux density, W/m^2; r, cylindrical coordinates, m; r_0, radius of a circular heat source, m; R, radius of a body, m; s, parameter of Laplace integral transformation θ_{Hs}; t, temperature, ^oC; t_{en}, initial temperature of a body, ^oC, K; z, Cartesian coordinates, m; $\vartheta = t - t_{en}$, excess temperature, ^oC; λ, thermal conductivity of a body, $W/(m \cdot K)$; τ, time, sec; φ, angle of inclination of the $\vartheta_c \to \sqrt{\tau}$ curve. Subscripts: en, environment; c, center.

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